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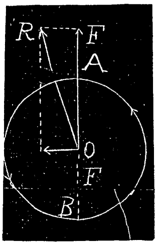
MISÇELLANEOUS.

77. **Proposed by T. E. COLE, Columbus, Ohio.**

It is said that a base-ball pitcher throws curves. Give a scientific explanation of how it is done.

Solution by WALTER H. DRANE, A. M., Graduate Student, Harvard University, Cambridge, Mass.

When a ball pitcher wishes to throw a curve, in addition to the onward motion given, he sets the ball to revolving rapidly about its own axis. Now the resistance the air offers to a moving body depends upon its velocity, and the only resistance besides gravity which the ball encounters in its motion is this resistance of the air. Consider the ball as revolving from right to left as in the figure, and let us regard it as two bodies, the line of division being the line of its own motion onward. The particles of the right half of the ball are moving around either directly in the same direction as the onward motion, or in oblique directions ; but these oblique velocities may all be resolved into two velocities. One parallel to and in the same direction *OF*, and one perpendicular to *OF*, which last has no effect on the onward velocity. Consequently the particles of the right half have, in addition to the onward velocity of the ball, a velocity due to their revolution about *O*, this increase of velocity depending upon the distance of a particle from the center. Now in exactly the same way it can be shown that the velocity of particles on the left is decreased in proportion as that of those on the right is increased. Hence the resistance of the air offered to particles on the sight is slightly greater than that offered on the left. The result is that in effect a backward force is brought into play on the right which does not act on the left. Resolve this force parallel and perpendicular to *OF*. That parallel to *OF* does not effect the direction of motion. But combining that component perpendicular to *F* with *F* itself, we get that the total effect is to change the direction of the onward velocity, and since both the onward velocity and the revolution of the ball about its axis change at every instant of time, this change in direction must vary at any instant, and hence the ball moves in a curved line to which *OR* is a tangent.



This is merely a case of constrained motion as the above shows.

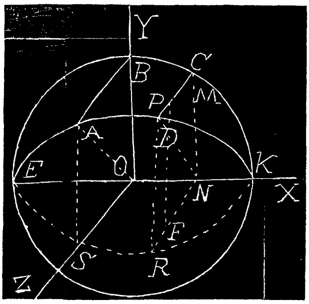
Also solved by *ALOIS F. KOVARIK.*

78. **Proposed by WALTER H. DRANE, A. M., Graduate Student, Harvard University, Cambridge, Mass.**

The center of a regular polygon of n sides moves along a diameter of a given circle, the plane of the polygon being perpendicular to the diameter, and its magnitude varying in such a manner that one of its diagonals always coincides with a chord of the circle; find the surface and the volume generated, and thence deduce the formulae for the surface and the volume of a sphere.

Solution by the PROPOSER.

Let *AB* be a side of the polygon in an initial position ; *CD* the side after it has moved an infinitesimal distance ON. Then $ABCD$ is an elementary portion of $1/n$th the required area, and the prism $AOB-DNC$ an elementary portion of $1/n$th the required volume. The equation of the plane $ADKO$ is $z=\cot(\pi/n).y$. Let the equation of the sphere of which the circle KBE is a great circle be



$$x^2+y^2+z^2=a^2.$$

Combining the equation of the plane and sphere gives $x^2+\sec^2(\pi/n)z^2=a^2$, which is the equation of $ESRK$, the projection on the plane xz of the curve of intersection of the plane and sphere. Now let P be any point in the area. $CN=\sqrt{(a^2-x^2)}$. $CM=MP\tan MPC=MP\tan(\pi/n)=z\tan(\pi/n)$.

$PF=y$; $CN=CM+PF$ or $\sqrt{(a^2-x^2)}=z\tan(\pi/n)+y$, which is the equation of the surface.

$$\frac{A}{2n}=\int\frac{\sqrt{\left[\left(\frac{df}{dx}\right)^2+\left(\frac{df}{dy}\right)^2+\left(\frac{df}{dz}\right)^2\right]}}{\frac{df}{dy}}$$

$$=\int_0^a\int_0^{\cos(\pi/n)\sqrt{(a^2-x^2)}}\frac{\sqrt{[a^2\sec^2(\pi/n)-x^2\tan^2(\pi/n)]}}{\sqrt{[a^2-x^2]}}dzdx$$

$$\cos(\pi/n)\int_0^a\sqrt{[a^2\sec^2(\pi/n)-x^2\tan^2(\pi/n)]}dx$$

$$=\frac{\sin(\pi/n)}{2}\left[a^2\cot(\pi/n)+(\pi a^2/n)\text{cosec}^2(\pi/n)\right].$$

$$\therefore\ A=na^2\cos(\pi/n)+\frac{\pi a^2}{\sin(\pi/n)}.$$

Evaluating this when $n=\propto$, by reducing to a common denominator, and then differentiating both numerator and denominator four times, this becomes $4\pi a^2$, the area of a sphere.

For volume we have,

$$V/2\pi=\sin(\pi/n)\cos(\pi/n)\int_0^a(a^2-x^2)dx=\tfrac{2}{3}a^3\sin(\pi/n)\cos(\pi/n).$$

$$\therefore\ V=\tfrac{4}{3}na^3\sin(\pi/n)\cos(\pi/n)=\tfrac{2}{3}na^3\sin(2\pi/n).$$

Throwing this in the typical form, and evaluating when $n=\infty$ by differentiating twice, gives $\frac{4}{3}\pi a^3$, the volume of a sphere.

Also solved by *G. B. M. ZERR*.